

The Percolation problem: Solution with smart simulations

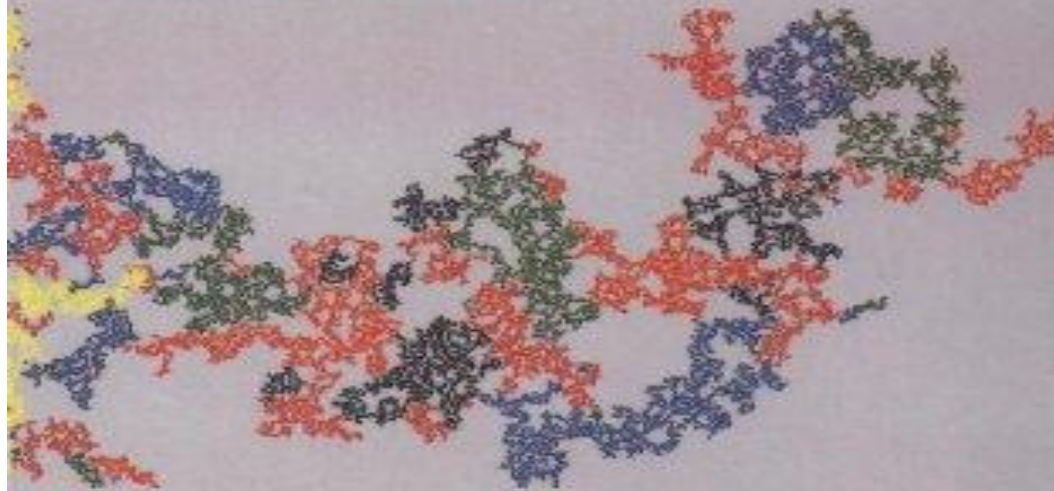
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The percolation problem



Introduction to **PERCOLATION THEORY**

Revised Second Edition

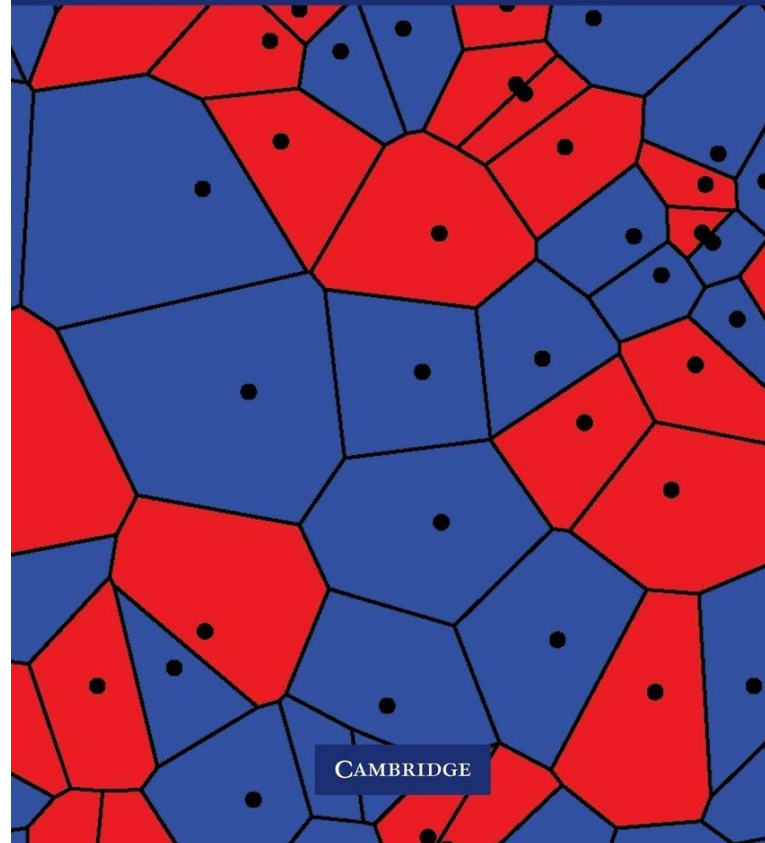


**DIETRICH STAUFFER AND
AMNON AHARONY**

PERCOLATION



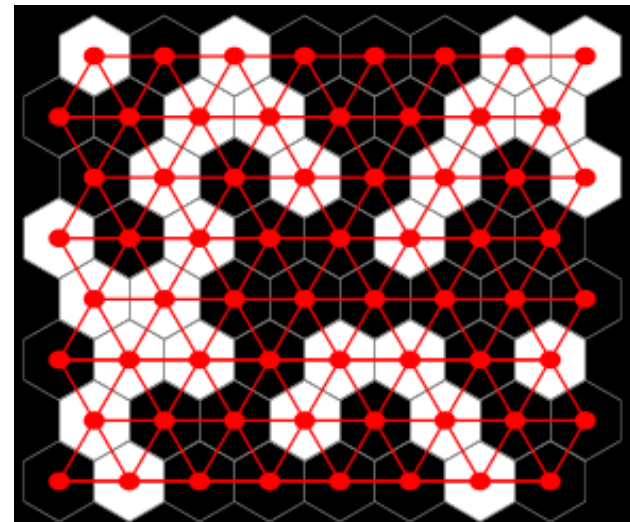
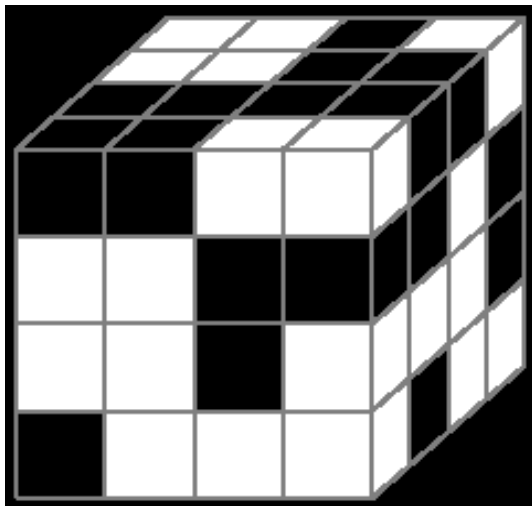
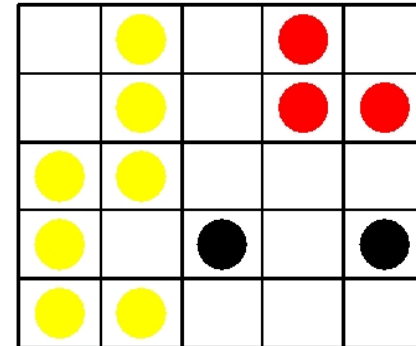
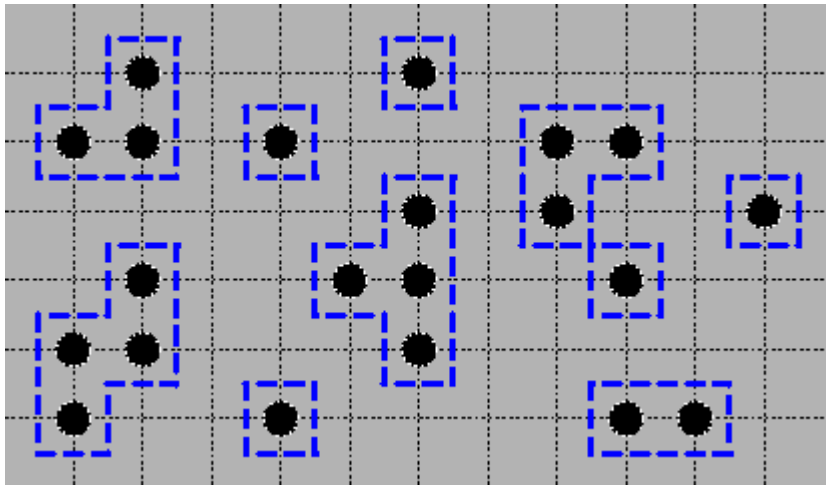
Béla Bollobás and Oliver Riordan



CAMBRIDGE

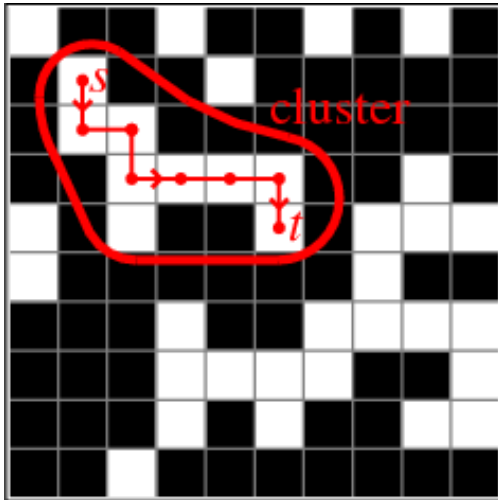
What is the problem?

- system made of 2 types of entities
- open/closed, true/false, conducting/insulating
- randomly mixed
- fixed ratio of open/closed, called “p”
- p in the range $0 < p < 1$
- adjacent entities of same type form clusters
- clusters depend on topology
- can be on lattice sites or on lattice bonds

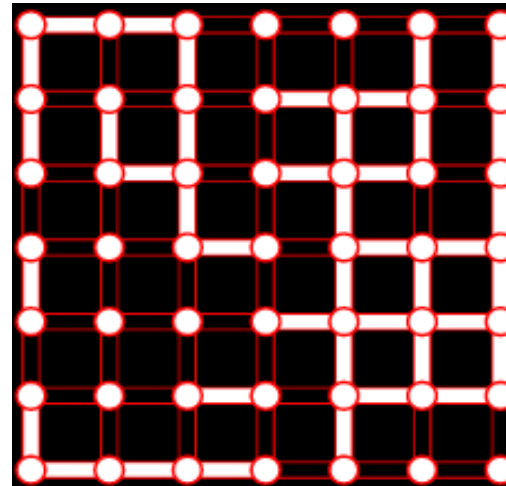


Site or bond percolation

site

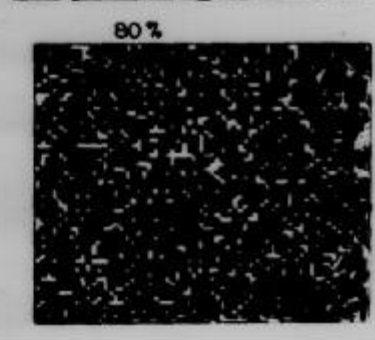
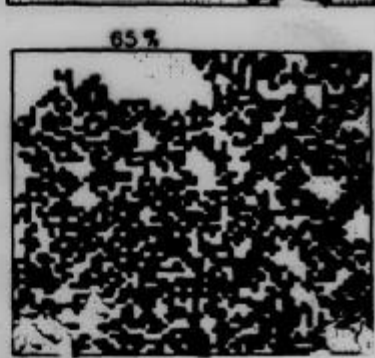
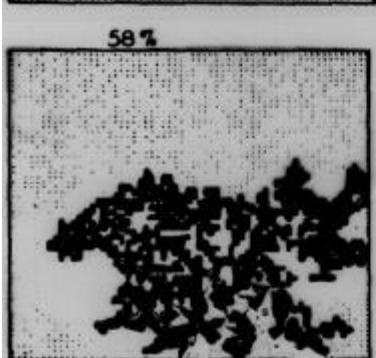
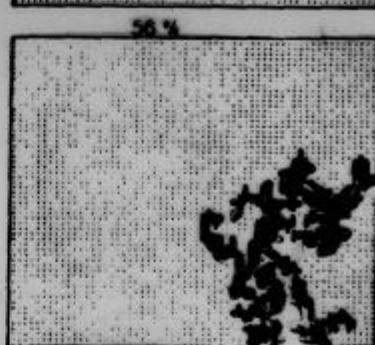
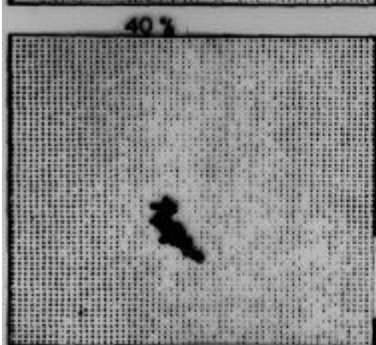
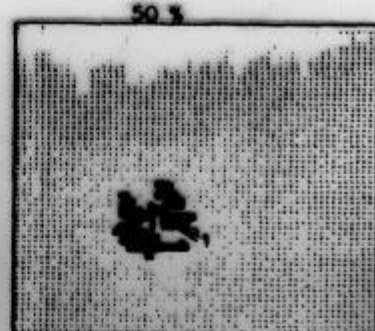
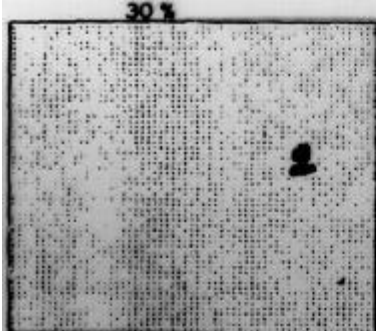


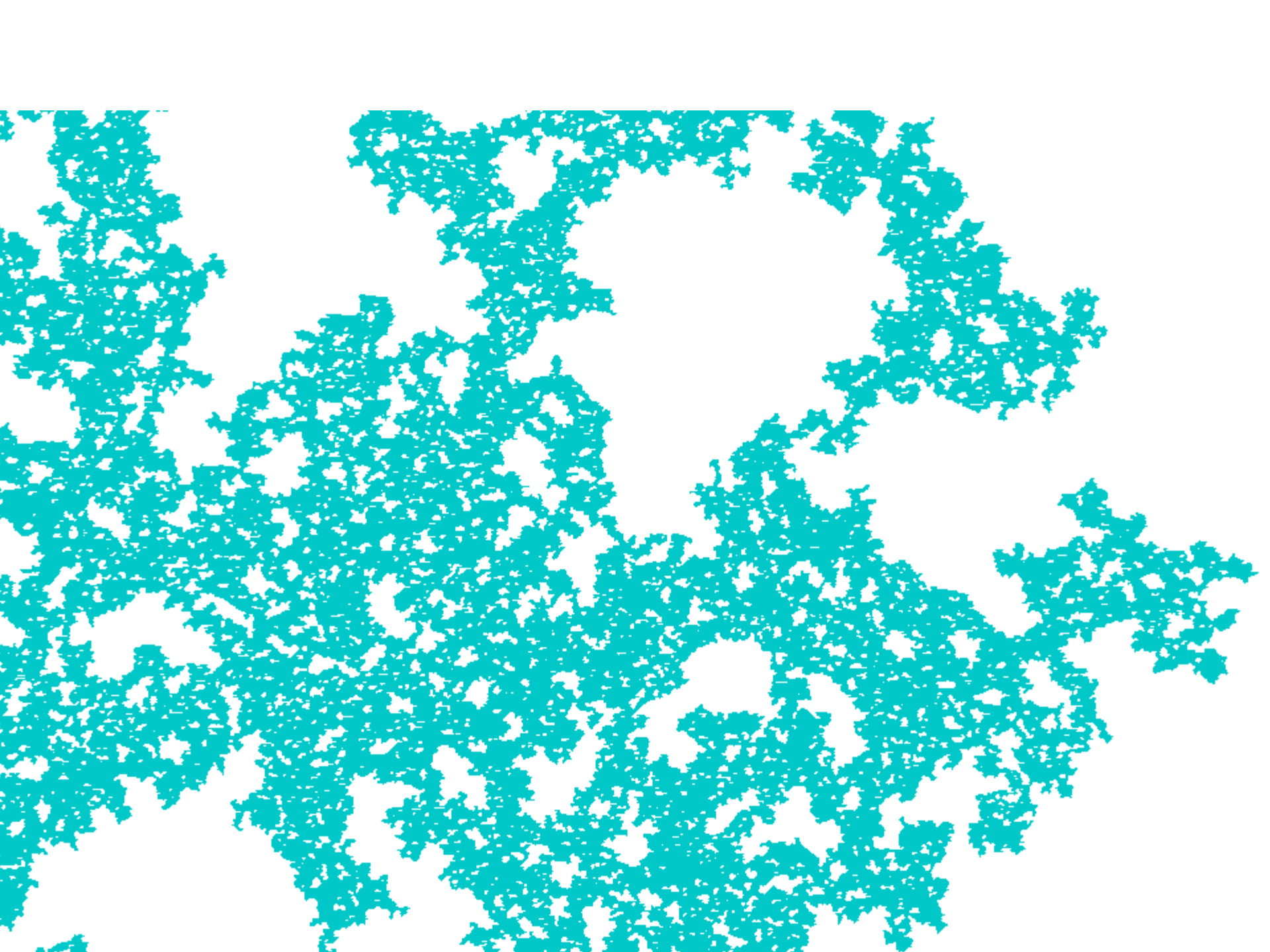
bond



Percolation phase transition

- focus on largest cluster only
- size increases abruptly at the critical point
- system goes through a phase transition from “insulating” to “conducting”
- 2nd order phase transition, $\Delta H=0$





Percolation simulation

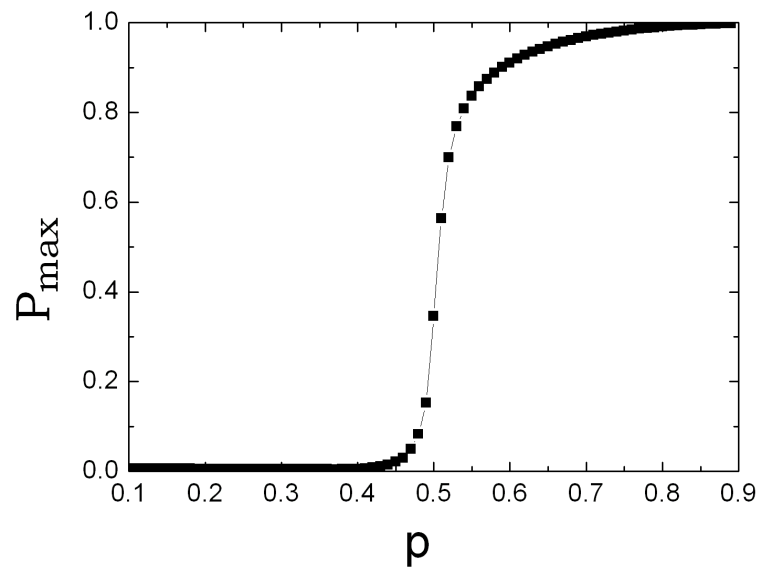
$$P_{\max}=\frac{m_{\max}}{pN^2}$$

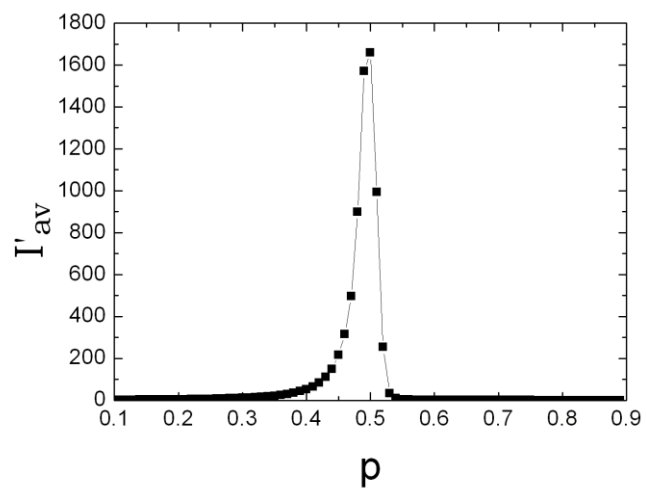
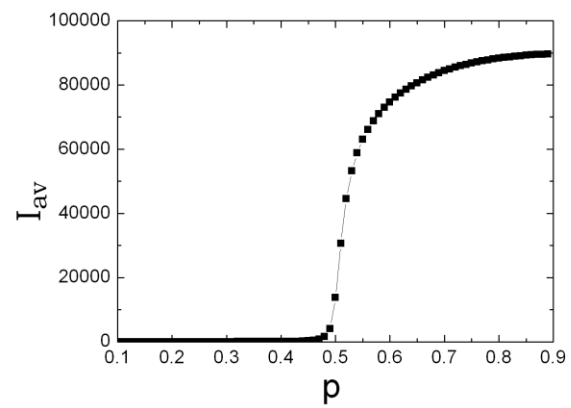
$$I_{\alpha v}=\sum_{m=1}^{m_{\max}}\frac{i_m m^2}{pN^2}$$

$$I_{av}'=I_{av}-\frac{i_{\max}*m_{\max}^2}{p*N^2}$$

$$I_{av}'=\sum_{m=1}^{m-m_{max}}\left(\frac{i_m\cdot m^2}{p\cdot N^2}\right)$$

$P(\max)$





How can we estimate p_c ?

- several techniques have been developed
- square lattice (site percolation) $p_c = 0.5927\dots$
- cannot be proven analytically
- square lattice (bond percolation) $p_c = 0.5000$
- simple cubic (site) $p_c = 0.3116\dots$
- simple cubic (bond) $p_c = 0.2488\dots$
- p_c strongly depends on the lattice type
- the more nearest neighbors, the lower the p_c

Cluster Multiple Labeling Technique (CMLT)

- sweep the lattice from one end to the other
- for every cluster that appears give a different index number
- everytime 2 clusters join, they become one cluster
- “brute force” method: go back and merge the index numbers of the 2 clusters into 1 index number only. Need to sweep entire lattice
- CMLT method: need only a single sweep for the same job
- Invented by Hoshen (1976), called Hoshen-Kopelman algorithm

What happens when 2 clusters coalesce

- we need to add the 2 sizes into 1
- we change the label of the index, but NOT the index itself

Before the joining:

$L(1)=1, L(2)=2, L(3)=3.....$

After joining:

$L(3)=2.....$

1	0	1	1	0	0	0	1	1	0	1	0	1	0	0	0	1	1	0	1
1	1	1	1	0	1	0	0	0	1	0	1	1	1	1	1	1	0	1	1
1	0	1	0	1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	0
0	1	1	1	1	0	0	1	1	1	1	0	1	1	1	1	1	0	1	1
0	0	0	0	1	1	1	1	0	1	0	1	1	0	0	0	1	0	1	1
0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	1	1	0
1	1	1	0	0	1	1	1	1	0	1	1	0	1	1	1	0	0	1	0
1	1	1	0	1	1	1	1	1	0	1	0	1	0	0	0	0	0	1	1
1	1	0	0	1	1	0	0	0	0	1	1	0	1	0	0	0	0	0	1
1	0	1	0	0	1	0	1	1	1	0	0	0	1	1	0	0	0	1	1
1	1	0	1	1	0	1	1	1	1	0	1	0	0	1	1	1	0	0	0
0	1	1	1	1	0	1	0	1	0	1	0	1	0	1	1	0	0	0	1
1	1	1	0	0	1	1	0	0	0	0	0	1	0	0	1	0	1	0	1
1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	0	1	0	0	1
0	1	0	1	1	0	1	1	1	1	1	1	1	1	1	0	0	1	0	1
0	1	0	1	1	1	1	0	1	1	0	1	0	1	0	1	0	1	1	1
1	0	1	1	0	0	0	1	1	1	1	0	0	0	1	1	0	1	1	1
1	1	1	0	0	0	1	0	1	0	1	0	0	1	0	0	1	1	0	1
0	1	1	1	0	0	1	0	0	1	0	0	0	1	0	1	1	0	1	0
0	0	1	0	1	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1

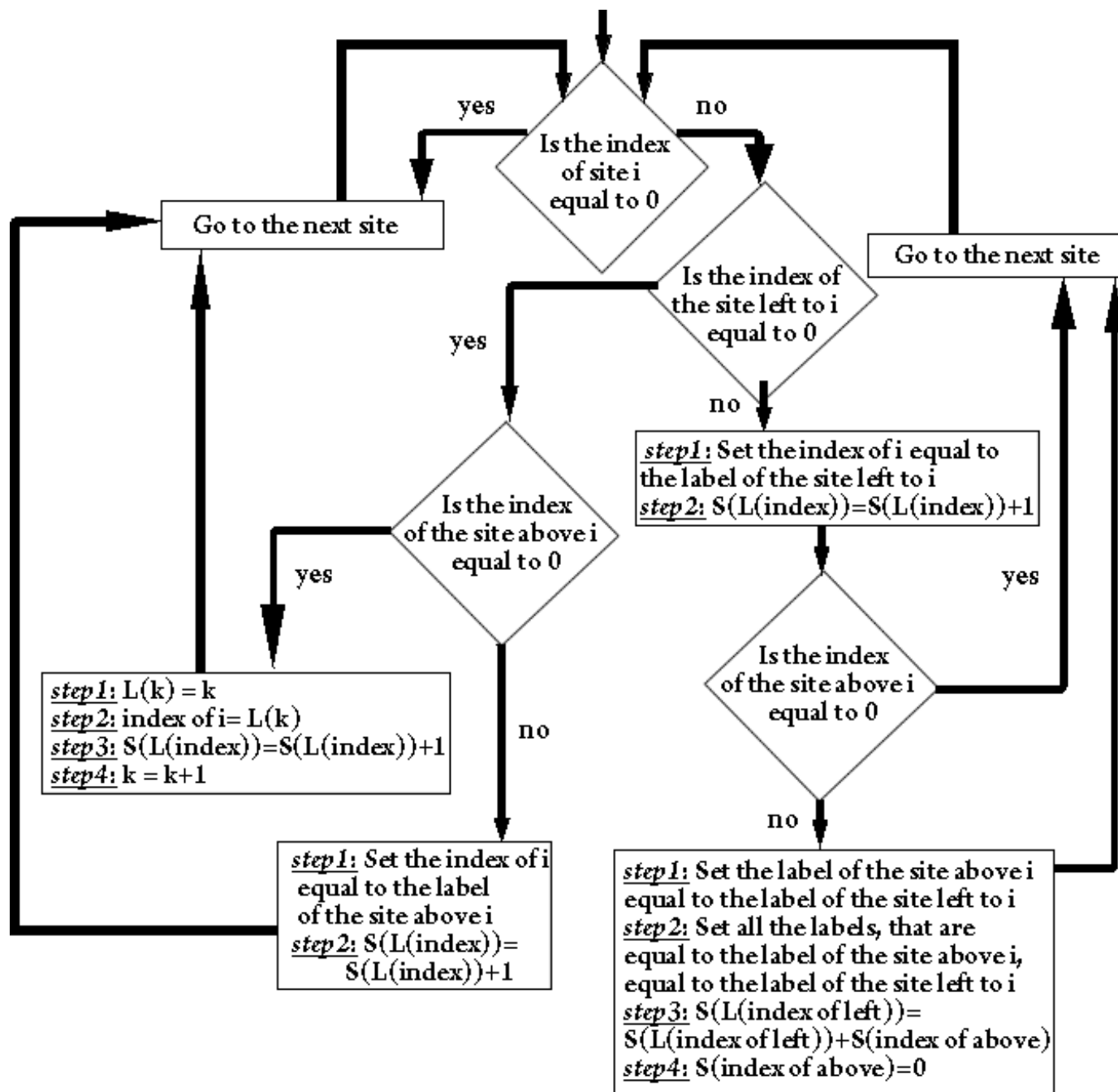
Part (a)

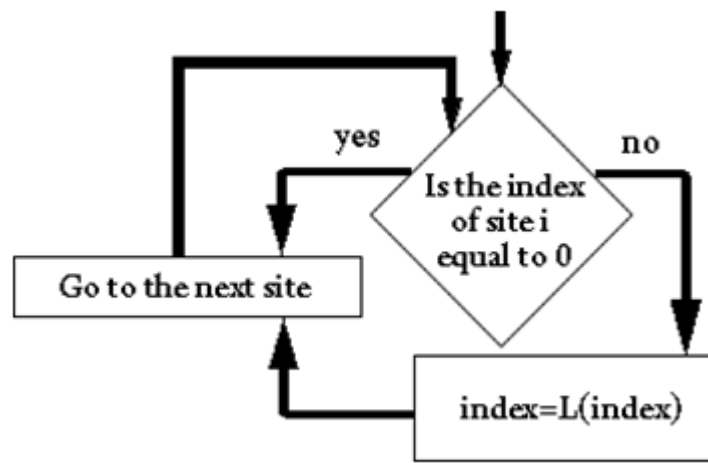
1	0	6	5	0	0	0	18	18	0	26	0	33	0	0	0	43	31	0	52
1	1	1	5	0	13	0	0	0	24	0	30	11	31	31	31	31	0	49	31
1	0	1	0	10	5	0	0	0	24	11	11	11	31	31	31	31	0	49	0
0	5	5	5	5	0	0	19	11	11	11	0	11	31	31	31	31	0	49	31
0	0	0	0	5	5	11	11	0	11	0	31	31	0	0	0	31	0	49	31
0	0	0	0	0	5	11	11	0	0	0	0	0	0	0	0	31	31	31	0
2	2	3	0	0	5	11	11	11	0	27	27	0	37	37	37	0	0	31	0
2	2	3	0	11	11	11	11	11	0	27	0	34	0	0	0	0	0	31	31
2	2	0	0	11	11	0	0	0	0	27	27	0	38	0	0	0	0	0	31
2	0	7	0	0	11	0	20	14	14	0	0	0	38	38	0	0	0	50	50
2	2	0	9	4	0	15	14	14	14	0	32	0	0	38	38	38	0	0	0
0	2	3	3	4	0	15	0	14	0	28	0	35	0	38	38	0	0	0	53
3	3	3	0	0	14	14	0	0	0	0	0	35	0	0	38	0	46	0	53
3	3	3	3	4	0	0	21	4	0	29	22	22	22	22	0	44	0	0	53
0	3	0	3	4	0	16	4	4	22	22	22	22	22	22	0	0	47	0	53
0	3	0	3	4	4	4	0	4	22	0	22	0	22	0	41	0	47	42	42
4	0	8	4	0	0	0	22	22	22	22	0	0	0	40	40	0	47	42	42
4	4	4	0	0	0	17	0	22	0	22	0	0	39	0	0	45	42	0	42
0	4	4	4	0	0	17	0	0	25	0	0	0	39	0	42	42	0	51	0
0	0	4	0	12	0	0	23	23	0	0	0	36	36	36	0	0	48	48	48

Part (b)

50	0	50	50	0	0	0	18	18	0	26	0	50	0	0	0	50	50	0	50
50	50	50	50	0	50	0	0	0	50	0	50	50	50	50	50	50	0	50	50
50	0	50	0	50	50	0	0	0	50	50	50	50	50	50	50	50	0	50	0
0	50	50	50	50	0	0	50	50	50	50	0	50	50	50	50	50	0	50	50
0	0	0	0	50	50	50	50	0	50	0	50	50	0	0	0	50	0	50	50
0	0	0	0	0	50	50	50	0	0	0	0	0	0	0	0	50	50	50	0
22	22	22	0	0	50	50	50	50	0	27	27	0	37	37	37	0	0	50	0
22	22	22	0	50	50	50	50	50	0	27	0	34	0	0	0	0	0	50	50
22	22	0	0	50	50	0	0	0	0	27	27	0	38	0	0	0	0	0	50
22	0	7	0	0	50	0	14	14	14	0	0	0	38	38	0	0	0	50	50
22	22	0	22	22	0	14	14	14	14	0	32	0	0	38	38	38	0	0	0
0	22	22	22	22	0	14	0	14	0	28	0	22	0	38	38	0	0	0	42
22	22	22	0	0	14	14	0	0	0	0	0	22	0	0	38	0	46	0	42
22	22	22	22	22	0	0	22	22	0	22	22	22	22	22	0	44	0	0	42
0	22	0	22	22	0	22	22	22	22	22	22	22	22	22	0	0	42	0	42
0	22	0	22	22	22	22	0	22	22	0	22	0	22	0	40	0	42	42	42
22	0	22	22	0	0	0	22	22	22	22	0	0	0	40	40	0	42	42	42
22	22	22	0	0	0	17	0	22	0	22	0	0	36	0	0	42	42	0	42
0	22	22	22	0	0	17	0	0	25	0	0	0	36	0	42	42	0	48	0
0	0	22	0	12	0	0	23	23	0	0	0	36	36	36	0	0	48	48	48

Part (c)





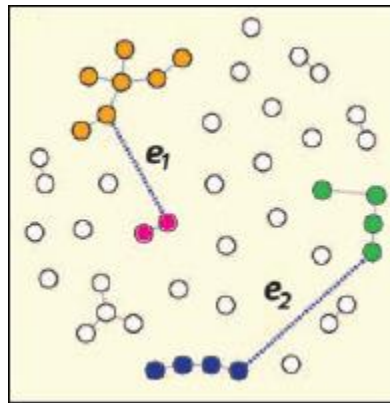
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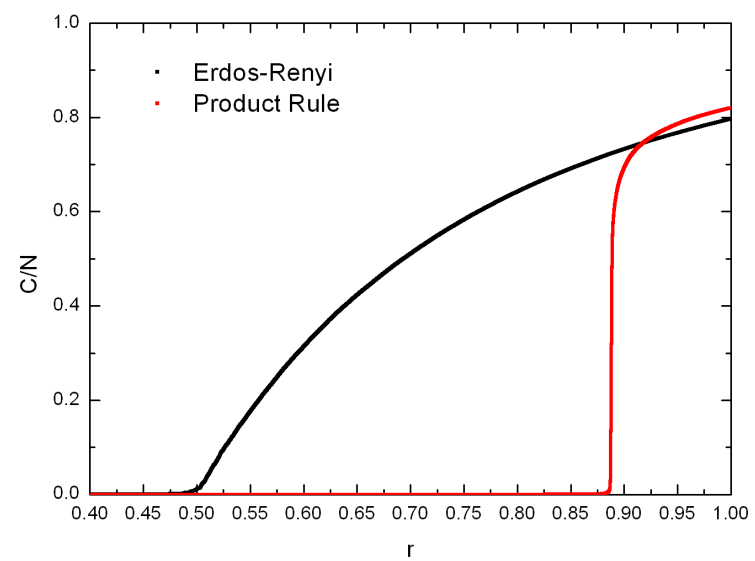
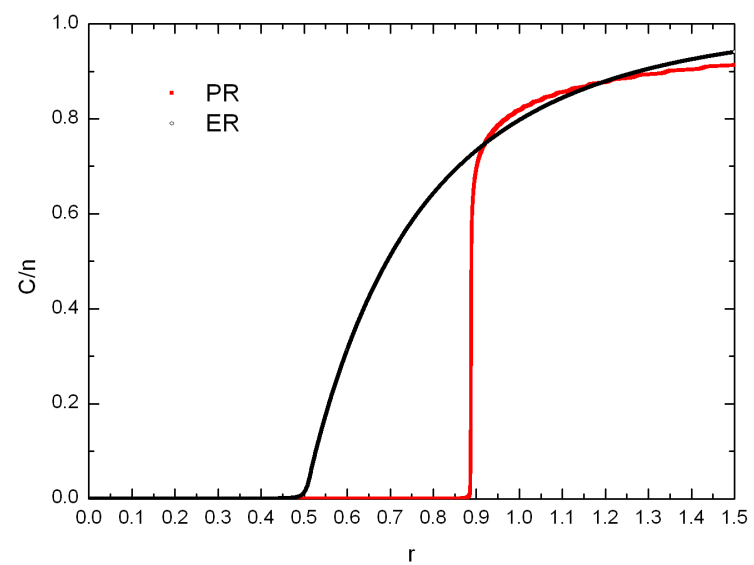
--->courses --->percolation

Achlioptas process

- developed in 2010
- new method of preparing the system
- use probe sites and fill lattice in such a way as to delay the criticality

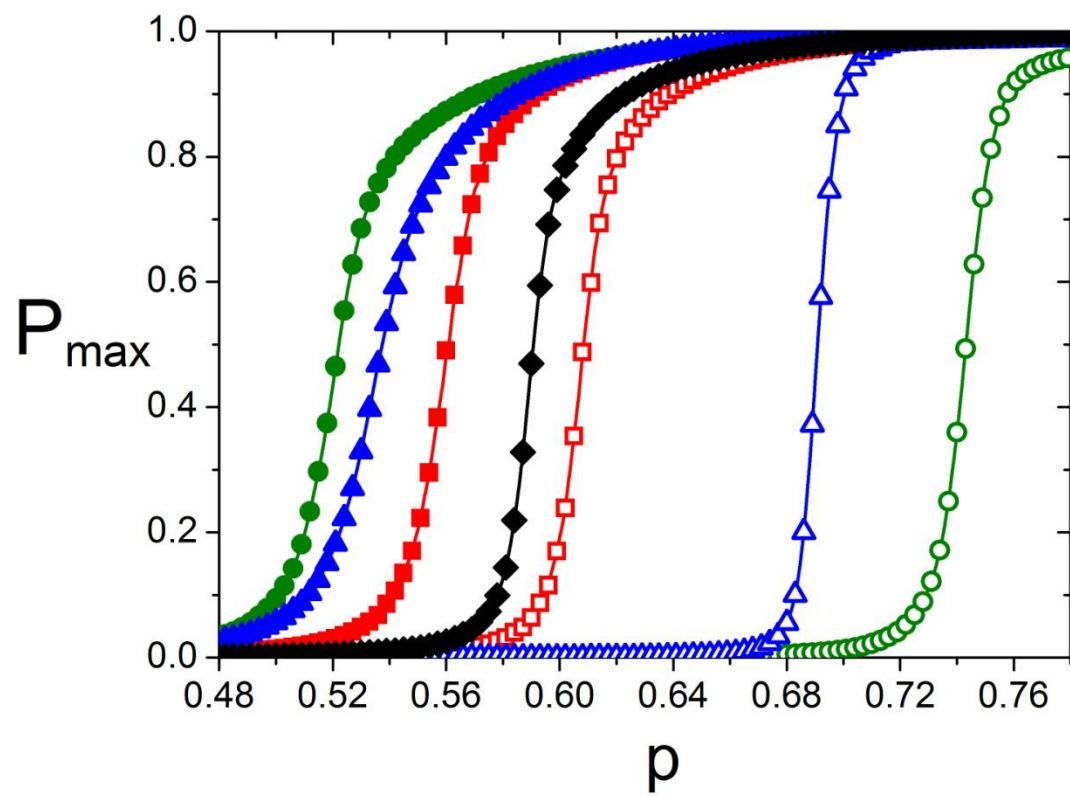
Achlioptas process - product rule

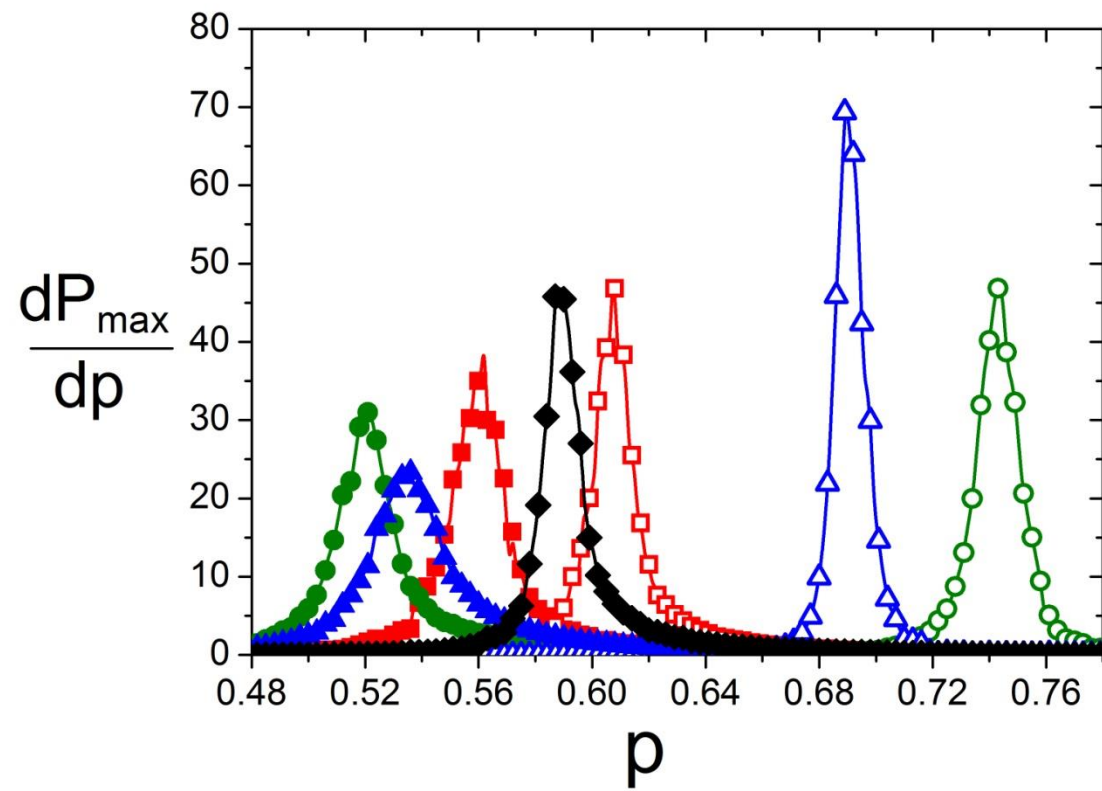




Many different variations

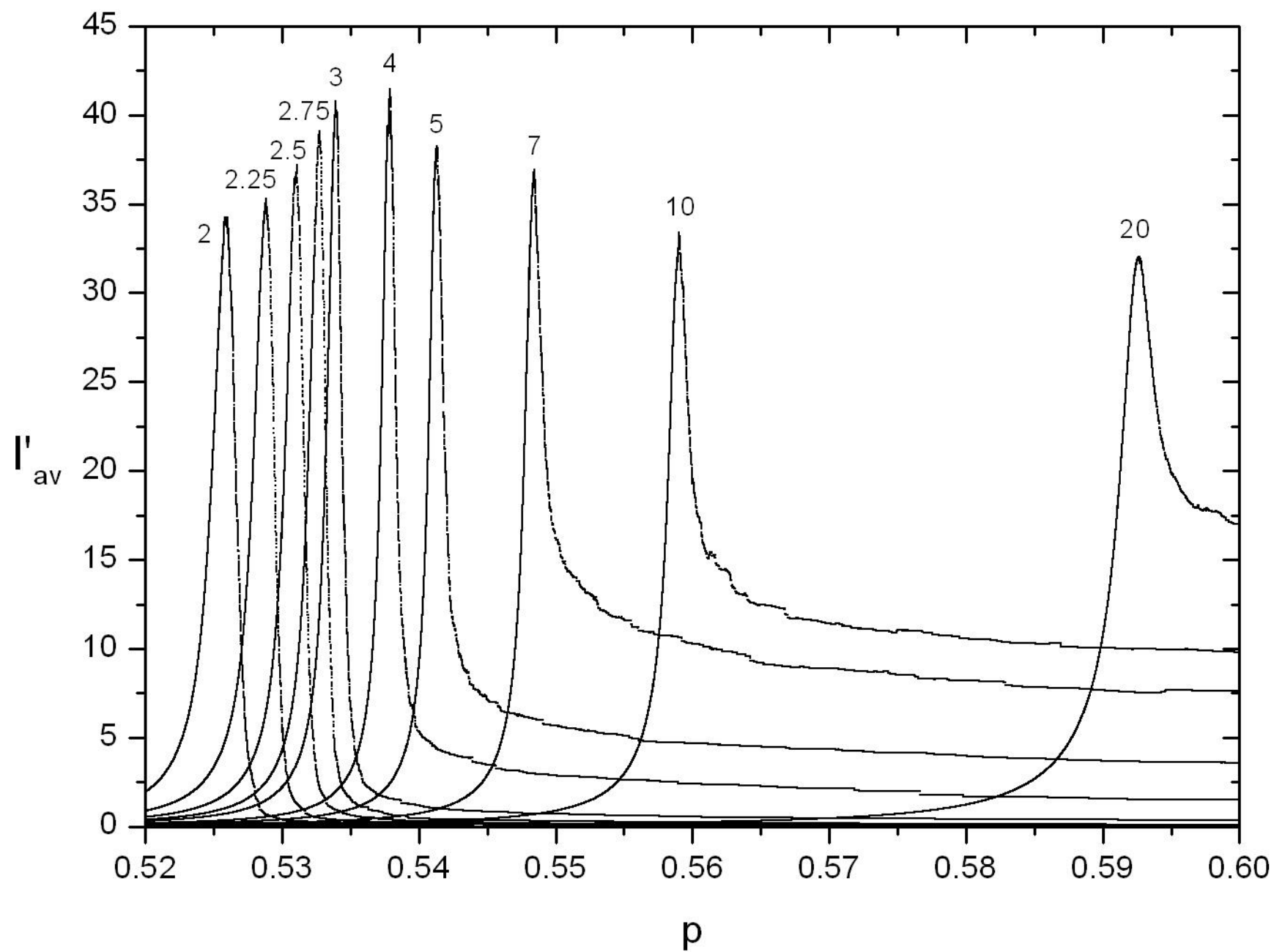
- Sum or product
- Allow the largest or the smallest
- Attraction or repulsion





Critical percolation threshold values

Model	p_c	d_f
Classical percolation	0.5927	1.89 ± 0.02
Attraction model ($k = 1$)	0.5618	1.89 ± 0.03
Repulsion model ($k = 1$)	0.6100	1.88 ± 0.03
Product rule (delay)	0.7554	1.99 ± 0.01
Product rule (early emergence)	0.5315	1.87 ± 0.02
Sum rule (delay)	0.6942	1.99 ± 0.01
Sum rule (early emergence)	0.5433	1.88 ± 0.02



Summary: percolation

- Old problem
- Most useful paradigm in phase transitions
(similar as Ising model)
- CMLT was first method, now many more
- Very useful in many-many different fields
- Problem is solved, but new variants emerge